(BEDT-TTF)$_2$Ag(CF$_3$)$_4$(TCE) has three polymorphs showing superconductivity, namely, the $\kappa_L$, $\kappa\alpha'_1$, and $\kappa\alpha'_2$-phases [1-3]. Although the first phase is a typical $\kappa$-type compound, the last two phases have recently been identified as dual-layered superconductors composed of the $\kappa$- and $\alpha'$-type donor layers. In the $\kappa\alpha'$-phases, the bond lengths of the donors indicate that the $\alpha'$-layers are likely in charge ordered states. We investigate the superconducting properties and Fermi surface of the $\kappa\alpha'_1$-phase. The temperature dependence of the resistance is similar to the typical $\kappa$-type BEDT-TTF salts, and the superconducting transition temperature (midpoint $T_c$) is 9.5 K. The interlayer coherence length is sufficiently smaller than the thickness of the conducting layer, indicating a two-dimensional superconductor. The de Haas-van Alphen (dHvA) oscillation shows that the conducting layer is the $\kappa$-layer with a half-filled band; this is in agreement with the charge-ordered $\alpha'$-layer due to the quarter-filled band. The effective cyclotron mass observed in the dHvA oscillation is larger than the bare cyclotron mass calculated. The BEDT-TTF superconductors including a $\kappa$-layer show the following trend: the higher the $T_c$ of the material, the larger the mass enhancement.